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Some Aspects of Liquid Movement in Phosphate Slime

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Abstract

The separation of liquid from solid is central to the disposal of some industrial effluents. This paper describes an exploratory investigation of self-weight filtration of phosphate slime produced during extraction of rock phosphate. It is found that many aspects of the process are quantitatively consistent with the theory of water movement in saturated swelling soils.

INTRODUCTION

Although there has been some examination of specific problems involving liquid and solid transfer in various swelling materials (*1–5*), it is only recently that an attempt has been made to examine systematically the general problem of the interaction of liquid and solid in swelling systems. In principle, the predictions that emerge from these studies are plausible, but because the material properties used in the illustrative calculations have, of necessity, been selected arbitrarily rather than experimentally (very few reliable experimental data exist), there remains some uncertainty about the precise behavior of the many swelling systems that occur in nature.

It is the purpose of this communication to explore a practically significant situation for which the theory of liquid flow in swelling systems should apply, to show that many aspects of the process are consistent with the theory, and to present experimentally derived data necessary to use the theory predictively.

It should be noted at the beginning that the work discussed here is restricted to two-phase (solid/liquid) situations. These are the most simple systems to examine experimentally and mathematically. They have an additional advantage in that since volumetric strain per unit change in liquid content is maximal (unity) in these systems, their use adequately tests the Langrangian basis of the flow theory as well as the consequences of mass transfer in the gravitational field.

Furthermore, it should be noted that the approach here differs significantly from that conventionally used to consider separation of liquid and solid in industrial effluents. In these situations in general, the approach is based on a consideration of the movement of solid particles relative to the liquid. Stokes' law is central to the analysis. The approach used here considers the flow of liquid *relative to the solid* in terms of Darcy's law. The interaction of particles thus presents no difficulties since it is described in terms of the suction/water-content relation and the hydraulic conductivity/water-content relation. The theory assumes no sorting of particles.

THEORY

If it can be assumed that both the water potential, Ψ, and the hydraulic conductivity, K, are single valued functions of the moisture ratio, ϑ (volume of water per unit volume of solid), then the vertical flow of water in a saturated swelling system is described by the nonlinear Fokker-Planck equation (*6–8*):

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial}{\partial m}\left(D_m \frac{\partial \vartheta}{\partial m}\right) + (1 - \gamma_c)\frac{dK_m}{d\vartheta}\frac{\partial \vartheta}{\partial m} \tag{1}$$

In this equation, t is time, γ_c is the specific gravity of the solid, $K_m = K/(1 + \vartheta)$, $D_m = K_m\, d\Psi/d\vartheta$, and m is a material coordinate defined (*1–3*, *5*) by

$$dm/dz = (1 + \vartheta)^{-1} \tag{2}$$

The vertical space coordinate, z, is defined positive upward. Equation (1) assumed that the volume flux density of liquid, v, relative to the solid particles, is described by Darcy's law, viz.,

$$v = -K\frac{\partial \Phi}{\partial z} \tag{3}$$

and that the total potential, Φ, of the water (per unit weight of water) is given by

$$\Phi = \Psi + z + \int_z^L \gamma \, dz \qquad (4)$$

In Eq. (4), L is the upper surface of the slurry and γ is its wet specific gravity.

Philip (*6*) explored, in principle, solutions of Eq. (1) for unsteady vertical flows, noting that mathematical details were likely to be elaborate and that the presence of $(1 - \gamma_c)$ in the "gravitational" term of Eq. (1) would have the result that infiltration into a swelling soil would be in some ways analogous to capillary rise in a nonswelling one. Smiles (*9*, *10*) showed experimentally that this latter contention of Philip is correct, but that in practical situations in which the conditions

$$\vartheta = \vartheta_n, \qquad m > 0, t = 0$$
$$\vartheta = \vartheta_0, \qquad m = 0, t > 0 \qquad (5)$$

appear to be realized, the gravitational term may be neglected for long periods of time. The observations that cumulative outflow, i, graphed with respect to $t^{1/2}$ is linear and that the profiles of $\vartheta(mt^{-1/2})$ preserved similarity were cited as evidence. The effect was attributed to the differences in the order of magnitude of D_m and $(1 - \gamma_c)\, dK_m/d\vartheta$. The observations result in great simplification of the solution of Eq. (1) subject to Conditions (5) since the problem then reduces to one analogous to sorption in a nonswelling system.

With the exception of one civil engineering example, these observations applied to bentonite pastes. More recently, however, experimentation has been extended to the red mud produced during the Bayer caustic-soda extraction of alumina from bauxite. This material presents problems because the volumes produced are great, and it is important that the liquor be separated as effectively as possible from the solid. In the first instance, the process of self-weight filtration was examined. Experimental data for both bentonite and red mud were published by Smiles (*8*) for this process. It was concluded that (a) Langrangian mathematics of swelling soils is appropriate to liquid flow in consolidating saturated slurries, (b) the early stages of self-weight filtration of slurries correspond formally to conventional Eulerian mathematics of sorption in nonswelling soils, and (c) the final equilibrium solid and liquid profiles are predictable using the definition of Φ of the water in a saturated swelling soil (Eq. 4).

For both these materials, D_m is about two orders of magnitude greater than $(1 - \gamma_c)\, dK_m/d\vartheta$. As a result, the analog of the sorption phenomenon is sustained.

In a further series of experiments to confirm these conclusions for other materials, some more complicated behavior was observed. These experiments were performed with so-called phosphate slimes produced during the extraction of rock phosphate. These materials also present problems to the industry from a liquid/solid separation point of view and because they are produced in vast quantities.

EXPERIMENTAL

In these experiments $\Psi(\vartheta)$ (Fig. 1) and $D_m(\vartheta)$ (Fig. 2) were determined using the method of Smiles and Harvey (*11*). Figure 3 shows $K_m(\vartheta)$ cal-

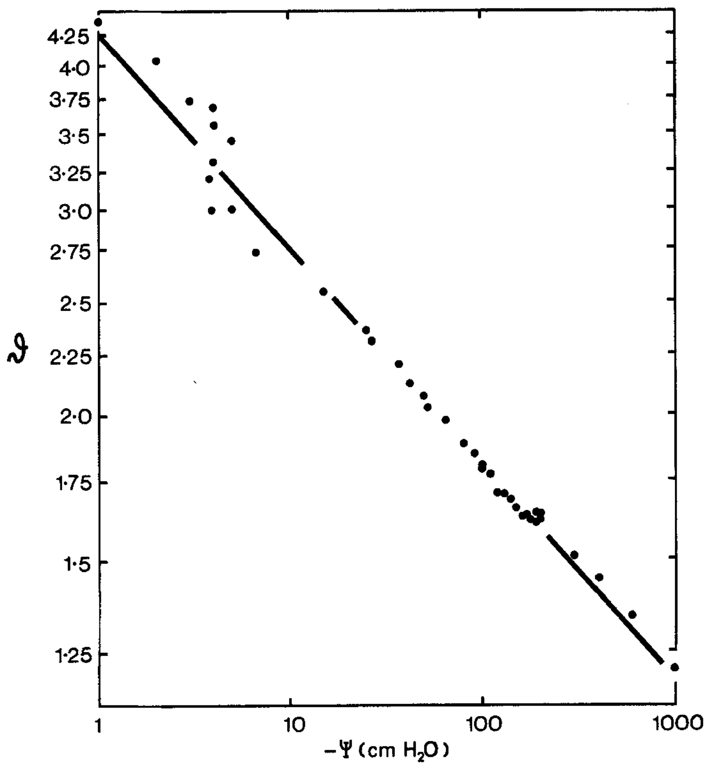

FIG. 1. Experimental data relating the moisture ratio, ϑ, in equilibrium with the moisture potential, Ψ, for a sample of phosphate slime.

culated using Figs. 1 and 2 and the equation $K_m = D_m\, d\vartheta/d\Psi$. $\gamma_c = 2.98$.

In addition, a self-weight filtration experiment was performed with a column of this material. The column was initially 30 cm long with a uniform moisture ratio of 18.2. It was permitted to drain at its base through a 0.45-μ Millipore membrane filter to a free water surface at the same height as the base. The cumulative outflow, i, was recorded as a function of time,

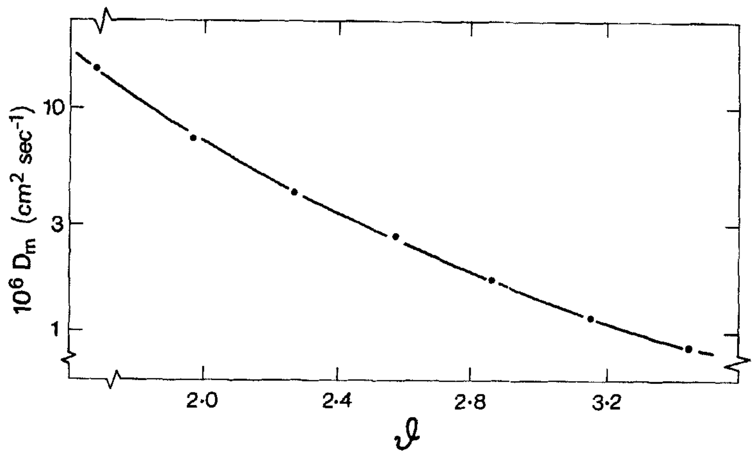

FIG. 2. Relationship between the moisture diffusivity, D_m, in Lagrangian coordinates, and the moisture ratio for a phosphate slime.

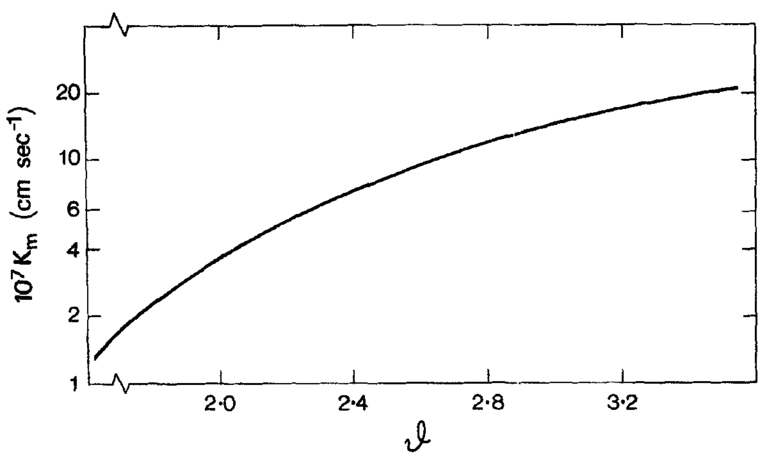

FIG. 3. The Lagrangian hydraulic conductivity, K_m, shown as a function of moisture ratio, ϑ. This curve is plotted using the data of Figs. 1 and 2.

t. Also, the height, h, of free water above the consolidating pad of solid material as well as the length, L, of this pad were recorded. Care was taken to minimize evaporation from the system. Figure 4 shows $i(t^{1/2})$ while Fig. 5 shows i, h, and L graphed as functions of time.

When the outflow rate was effectively undetectable, the solid pad was sectioned and the moisture/solid profile determined. Figure 5 shows this profile together with that predicted from $\Psi(\vartheta)$ (Fig. 1), assuming the column to have reached static equilibrium, using the method described by Smiles (*8*).

DISCUSSION

Figures 1, 2, and 3 reveal behavior that is qualitatively very similar to corresponding relations of bentonite (*11*) and red mud. Figure 1 shows that for this material $\vartheta(-\Psi)$ has the form $\vartheta = \alpha(-\Psi)^{\beta}$ with α and β constants. Data given in Ref. *12* has a similar form, but neither bentonite nor red mud are linear in a log-log graph. There appears to be no way to predict these relations.

Considering Fig. 2, D_m appears to decrease with increasing ϑ and does not exhibit the maximum described in Ref. *5* and predicted in Ref. *13* for

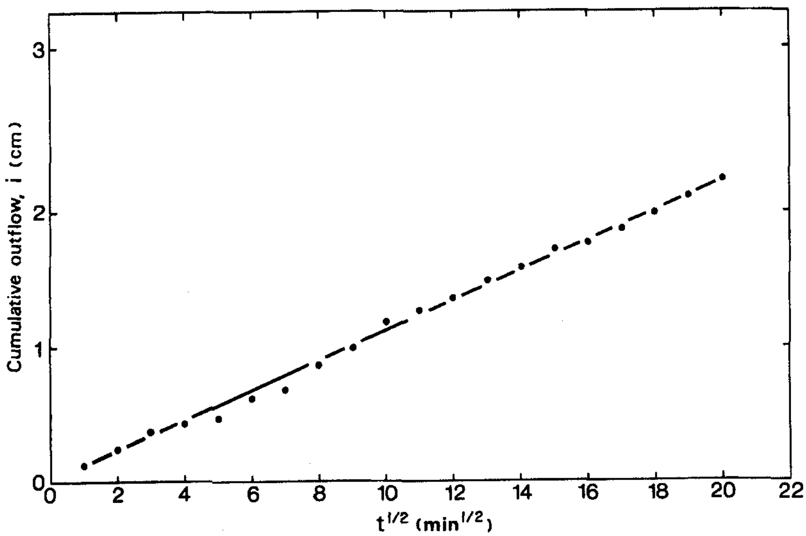

FIG. 4. The cumulative outflow from the base of the column, i, presented as a function of the square root of time, $t^{1/2}$.

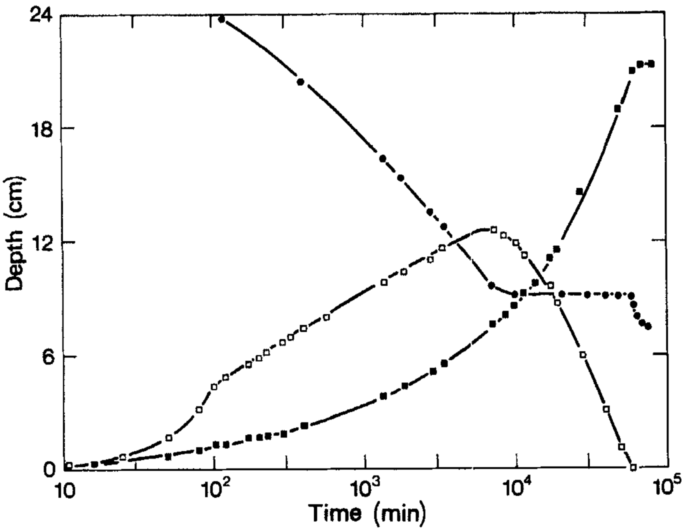

FIG. 5. Figure showing various aspects of self-weight filtration of a phosphate slime. Sediment (●) shows the way the sediment thickness, L, changes with time; ponded water depth (□) shows the depth of "clear" water, h, standing above the sediment; and cumulative outflow (■), i, refers to the cumulative volume of water that is collected after passing through the filter supporting the column.

saturated systems. One would, however, anticipate that as the material density increases, the swelling system will become more similar to a nonswelling material, air entry should occur, and D_m should decrease with decreasing ϑ. It is worth noting that the increase in D_m with decreasing ϑ is consistent with the typical outflow behavior of these swelling systems, shown, for example, in Ref. *11*, where $i(t^{1/2})$ is linear over most of the outflow period and then $di/dt^{1/2}$ rapidly becomes zero. The phenomenon corresponds to the course of sorption in a finite column of a nonswelling porous material where the diffusivity is an increasing function of the water content.

As might be expected, K_m increases with ϑ.

It should be noted here that since the calculation of D_m and subsequently K_m involve differentiation of experimental data, the errors involved may be significant. In this instance, however, the errors appear not to be great; K_m values of Fig. 3 are consistent with data from the filtration experiment, as discussed below.

Comparing Figs. 2 and 3, it will be noted that D_m is approximately 20 times greater than $-(1 - \gamma_c)\, dK_m/d\vartheta$ when $\vartheta = 1.6$, they are numerically approximately equal when $\vartheta = 2.4$, and when $\vartheta = 3.4$, $D_m \approx -0.2(1 - \gamma_c)\, dK_m/d\vartheta$. It would therefore be unwise to neglect the effect of the gravitational term in Eq. (1) when working with this material. The results shown in Fig. 4 confirm this. The graph of $i(t^{1/2})$ is linear for about 400 min, after which the rate of outflow declines although $i(t)$ never approaches linearity. The apparent sorptivity during this early period is 0.11 cm $\text{min}^{-1/2}$ and agrees reasonably well with a value of 0.09 cm $\text{min}^{-1/2}$ obtained in the outflow experiments used to determine D_m.

Examining now Fig. 5 and first the way the pad thickness, L, changes: the decrease in L occurs because of consolidation (corresponding to i) at the bottom and because of sedimentation at the top. Initially, these processes occur independently, but after about 9×10^3 min a steady-state profile appears to be established, at the upper surface of which $\Psi = 0$. The system then appears to behave as a falling-head permeameter. Figure 6 shows $\log(L + h)$ vs t. This relation is evidently linear and indicates a mean conductivity, $\bar{K} = 2.6 \times 10^{-6}$ cm/sec. Bearing in mind the likely errors in the calculation of K_m and K, this value agrees quite well with the range of K_m shown in Fig. 3.

In the concluding stages ($t > 6 \times 10^4$ min), no free water stands above the consolidating pad and drainage accompanied by further consolidation of the pad occurs. This final stage will be extended because of the low conductivity and vanishingly small gradients of potential in the pad. The

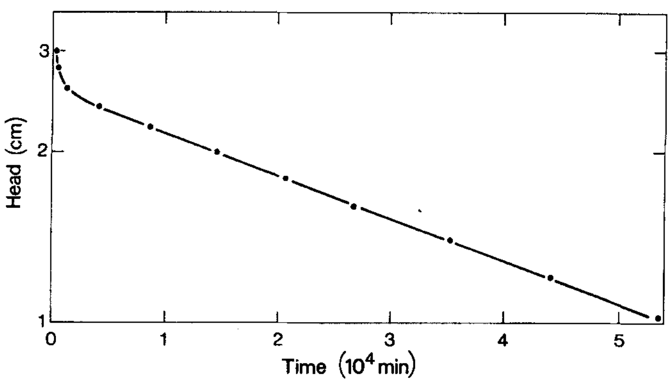

FIG. 6. Variation of hydrostatic head with time for the period of outflow when the sediment column was effectively of constant length.

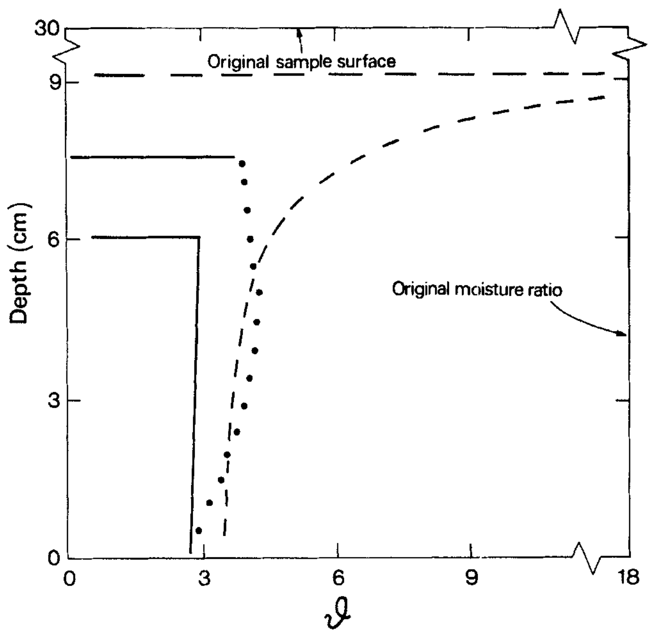

FIG. 7. Equilibrium moisture ratio profiles predicted for this experiment for sedimentation (- -) and self-weight filtration (——), together with the observed profile (●) for self-weight filtration after 7.5×10^4 min.

experimental data of Fig. 7 show that the dewatering process is, however, 90% complete if the calculated equilibrium profile is correct. Figure 5 also shows an equilibrium moisture distribution that would be observed if the column had been permitted to sediment without drainage from beneath. As might be expected, it is wetter throughout than the filtered column. Self-weight filtration should therefore give a greater recovery of liquid than sedimentation in slurries of equal initial depth and liquid content. This work continues and will be reported later.

Referring finally to the rate of accumulation of water above the settling pad: this appears to be a problem of sedimentation and one for which no adequate theory yet exists. It should be noted, however, that were the problem to be considered in terms of Eq. (1), then the early stages might be examined as flow in the case where

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial \vartheta}{\partial x} = 0, \qquad \text{i.e., } v = -K_m(1 - \gamma_c)$$

For phosphate slime, the very early stages ($0 < t < 1 \times 10^2$ min) show dh/dt to be effectively constant, giving $K_m = 3.9 \times 10^{-4}$ cm/sec. This may be a realistic estimate for K_m at $\vartheta = 18.2$ and raises the question—might it not be useful to use the Stokesian rate of fall as an indication of a maximum conductivity of a uniform porous material with a high moisture content?

It is concluded that many important aspects of self-weight filtration of phosphate slime are quantitatively consistent with the theory of water movement in saturated swelling soil and to this extent serve to confirm the theory. It should, at the same time, be noted that the experiment described here was essentially exploratory and is to be followed by others to examine the application of Darcy's law and the Lagrangian continuity equation to predict all aspects of one-dimensional flow in systems that can be described in terms of a hydraulic conductivity/water-content relation, a water potential/water-content relation, and in which the solid density is known.

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